

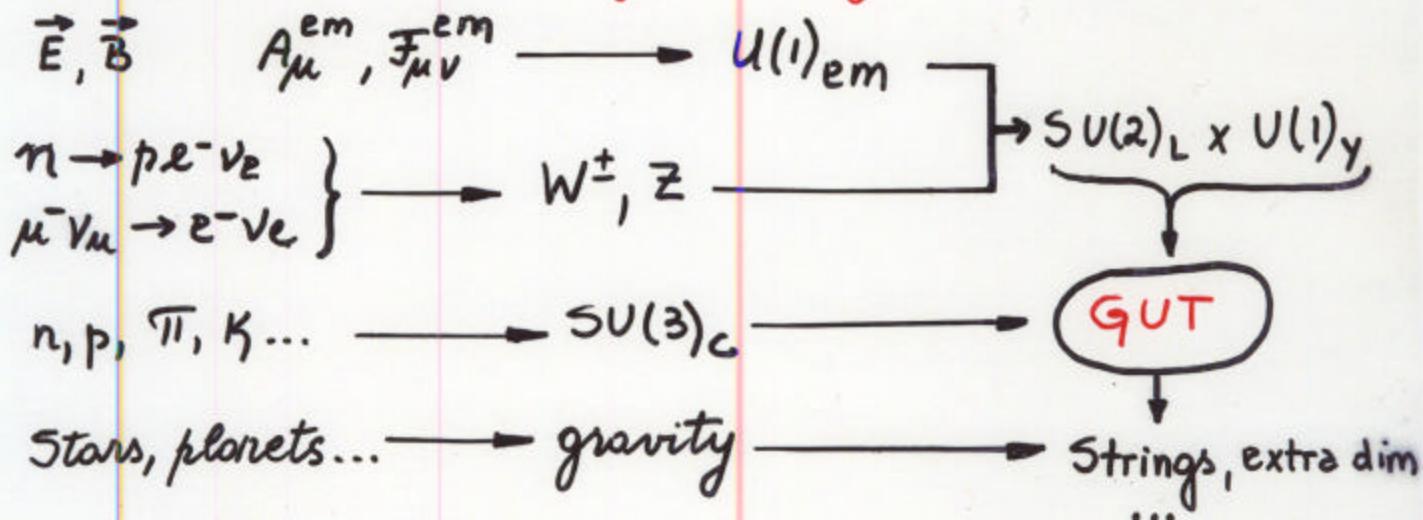
Fundamental Reasons to go Beyond the SM

- understand the origin of the symmetry group

Why $SU(3)_c \times SU(2)_L \times U(1)_Y$?

↑
hypercharge assignment (?)

- find a unified description of all particle interactions including gravity?



- understand the origin of the many free param. of the model

no prediction for Higgs & fermion masses

$$m_H^2 = 2 \tilde{\rho} v^2 \quad m_f = h_f v \quad \tilde{\rho}, h_f \rightarrow \text{free param}$$

($g_i, v = 174 \text{ GeV} \longrightarrow M_W, M_Z, \Gamma's.$)

- find a solution to the hierarchy problem of the SM Higgs sector

↓
instability of m_H under radiative corrections

Hierarchy Problem of the Higgs Sector of the SM

since the SM is an effective theory, one would expect low energy quantities (couplings & masses) to be given as a fc. of param of the fundamental theory valid at scales $Q > \Lambda_{\text{eff}}^{\text{SM}}$

$$g_i(\Lambda_{\text{eff}}^{\text{SM}}), \beta(\Lambda_{\text{eff}}^{\text{SM}}), h_f(\Lambda_{\text{eff}}^{\text{SM}}), m^2(\Lambda_{\text{eff}}^{\text{SM}}) \leftarrow \text{param in } V(\phi)$$

↓ → using RG evolution
low energy values (radiative corrections)

- dimensionless couplings → receive $\log(\Lambda_{\text{eff}}^{\text{SM}})$ correc.

- what about m^2 ? $V(\phi) = -m^2\phi^+\phi + \beta(\phi)/2(\phi^+\phi)^2$

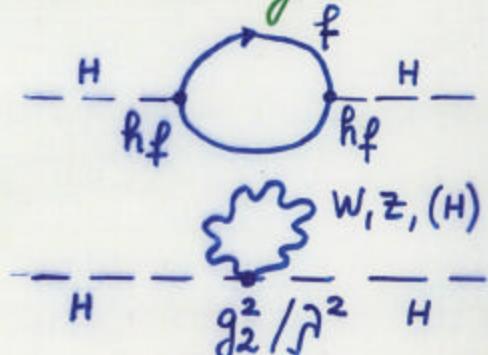
important since, $v^2 = m^2/\beta$

give masses to fermions, gauge bosons and $m_H^2 = 2\beta v^2$

Quantum corrections to m^2 are quad. divergent

$$m^2 = m^2(\Lambda_{\text{cutoff}}) + \Delta m^2$$

$$\Delta m^2 \sim \frac{n_W g_2^2 + n_H \beta^2 - n_f h_f^2}{16\pi^2} \Lambda_{\text{cutoff}}^2$$



to explain $v \approx 0(M_W)$

either $\Lambda_{\text{cutoff}} \lesssim 1 \text{ TeV}$ or extreme fine tuning to give unnatural cancellation between $m^2(\Lambda_{\text{cutoff}})$ & $\Lambda_{\text{cutoff}}^2$

The Naturalness \longleftrightarrow hierarchy problem of the SM
 → solution demands $\Lambda_{\text{eff}}^{\text{SM}} \lesssim 1 \text{ TeV}$
 two possibilities for the theory beyond $\Lambda_{\text{eff}}^{\text{SM}}$

i) with no fundamental scalars

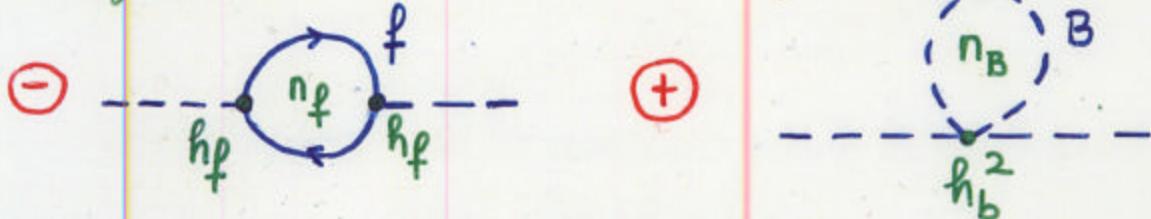
\implies Higgs field is a condensate of fermions

New strongly interacting theory at a scale $\Lambda_N \sim v$

\implies Technicolour Models

ii) with fundamental scalars

to cancel quad. divergences at scales $Q \gtrsim \Lambda_{\text{eff}}^{\text{SM}}$
 one needs a relation between the couplings
 to the Higgs of fermions and scalars $\rightarrow h_f = h_B$
 + also between the number of degrees of freedom
 of fermions & scalars $\rightarrow n_f = n_B$



Symmetry relating bosons & fermions

\implies SUPERSYMMETRY

Models with no fundamental scalars (Technicolor)

Higgs → composite of fundamental fermions

$$H \propto \bar{4}_R \cdot 4_L \Rightarrow \langle H \rangle \propto \langle \bar{4}_R \cdot 4_L \rangle$$

a mechanism like this occurs in QCD

<strong interactions between quarks and antiquarks lead to generation of mass, even starting with massless particles

$$\langle \bar{u}u \rangle = \langle \bar{d}d \rangle \neq 0$$

where $\bar{u}u = \bar{u}_L u_R + \bar{u}_R u_L$ $\bar{d}d$ analogously

since only left handed fields transform under $SU(2)_L \Rightarrow$ v.e.v above implies $SU(2)_L$,

in fact it breaks $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$

and gives mass to the weak gauge bosons

However, $\langle \bar{u}u \rangle \sim \langle \bar{d}d \rangle \sim \mathcal{O}(1_{QCD})^{d_3}$

and $\langle H \rangle = v \approx 174 \text{ GeV}$ observed in nature is much larger

Solution: New fermions $D_R, U_R, \begin{pmatrix} U \\ D \end{pmatrix}_L$ with a new strong interaction (strong at scale $\Lambda \approx v$) and asymptotically free: Scale up QCD or Technicolour

- Number of Bosons \longleftrightarrow Number of Fermions

- A particle and its SUSY partner have equal mass and couplings \iff Exact SUSY

Higgs mass corrections

$$(\Delta m^2)^f = -h_f^2 \int_{m_H^2}^{\Lambda} \frac{d^4 k}{(2\pi)^4} \frac{T_F \left[K\left(\frac{1+\gamma_5}{2}\right) K \right]}{(k^2 + m_f^2)^2}$$

$$\implies -\sum_f \frac{h_f^2}{16\pi^2} \left(\Lambda^2 - m_f^2 \ln\left(\frac{\Lambda^2}{m_H^2}\right) \right)$$

$$(\Delta m^2)^B = h_f^2 \int_{m_H^2}^{\Lambda} \frac{d^4 k}{(2\pi)^4} \left[\frac{1}{(k^2 + m_{f_L}^2)} + \frac{1}{(k^2 + m_{f_R}^2)} \right]$$

$$\implies \sum_B \frac{h_f^2}{16\pi^2} \left(\Lambda^2 - m_B^2 \ln\left(\frac{\Lambda^2}{m_H^2}\right) \right)$$

cancellation of quad. divergence has to do with relation between couplings (at Λ) & with $n_B = n_f$ but not with exact mass values

SUSY is broken in nature \leadsto extra mass to sfermion

$$\Rightarrow \Delta m^2 \sim h_f^2 \left[m_f^2 - m_{\tilde{f}}^2 \right] \ln\left(\frac{\Lambda^2}{m_H^2}\right) \sim h_f^2 M_{\text{SUSY}}^2 \ln\left(\frac{\Lambda^2}{m_H^2}\right)$$

No problem if $M_{\text{SUSY}} \approx 0(1 \text{ TeV})$

Beyond the SM

SM → effective theory up to M_{Planck} or below (?)

Precision measurements suggests that:

any new physics should

- lead to small correc. to electroweak observables
- be consistent with a light Higgs
- do not lead to large contrib. to FCNC

if possible, have the following good features

- lead to unification of couplings (Unified theory)
- incorporate gravity
- lead to a technical solution to the hierarchy problem (why $M_{\text{Pl}} \gg M_{\text{weak}}$)

Supersymmetry → fulfills all these properties

$$\text{if } \Lambda_{\text{eff}}^{\text{SM}} \simeq 1 \text{ TeV}$$

other solutions may also be possible

- Strong dynamics
- Extra Dimensions (w/ or without SUSY)

Supersymmetry

- Supersymmetric transformations relate bosonic to fermionic degrees of freedom
 \Rightarrow UNIFIED DESCRIPTION OF ELEMENTARY
 BOSONS AND FERMIONS IN NATURE

$$\begin{array}{ccc} F & \xrightarrow{Q_\alpha} & B \\ B & \xrightarrow{\bar{Q}_\dot{\alpha}} & F \end{array}$$

$Q_\alpha \rightarrow$ SUSY generator
 \downarrow
 2-comp. spinor

$$\{ Q_\alpha, Q_\beta \} = 0$$

$$P_\mu = (H, \vec{P}) ; G_\mu = (I, \vec{G})$$

$$\{ Q_\alpha, \bar{Q}_{\dot{\beta}} \} = G_{\alpha\dot{\beta}}^\mu P_\mu$$

Pauli matrices

\Rightarrow Supersymmetric Algebra naturally
 includes coordinate transformations

\equiv A LOCALLY INV. SUSY THEORY

YIELDS A THEORY INV. UNDER

COORDINATE TRANSFORMATIONS

Local SUSY \leftrightarrow SUPER GRAVITY

Supersymmetric $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$

→ simplest generalization of the spectrum

| | Bosons | Fermions | $SU(2)$ | Y |
|-------------------|--|--|--------------------|--------|
| Gauge Multiplets | $V^a \} W^\pm, Z$ $V^i \} \tilde{\chi}$ | $\tilde{J}^a \} \tilde{W}^\pm, \tilde{Z}$ $\tilde{J}^i \} \tilde{\chi}$ | triplet singlet | 0 0 |
| Matter Multiplets | $\tilde{L}^i = \begin{pmatrix} \tilde{\nu}_e \\ \tilde{e} \end{pmatrix}_L$ | $(\nu_e)_L$ | doublet | -1 |
| | $\tilde{E} \equiv \tilde{R} = \tilde{e}_R^*$ | e_R^c | singlet | 2 |
| | $\tilde{Q}^i = \begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}_L$ | $(u)_L$ | doublet | $1/3$ |
| | $\tilde{U} = \tilde{u}_R^*$ | u_R^c | singlet | $-4/3$ |
| | $\tilde{D} = \tilde{d}_R^*$ | d_R^c | singlet | $2/3$ |
| | $H_1^i = \begin{pmatrix} H_1^+ \\ H_1^- \end{pmatrix}$ | $\tilde{q}_{H_1}^i = \begin{pmatrix} \tilde{H}_1^+ \\ \tilde{H}_1^- \end{pmatrix}$ | doublet | -1 |
| | $H_2^i = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$ | $\tilde{q}_{H_2}^i = \begin{pmatrix} \tilde{H}_2^+ \\ \tilde{H}_2^0 \end{pmatrix}$ | doublet | 1 |

$$a = 1, 2, 3 \rightarrow SU(2) \text{ generator} \quad T^a = \sigma^a / 2; \quad Q = T^3 + Y/2$$

$$i = 1, 2 \rightarrow SU(2) \text{ indices}$$

QCD → g, \tilde{g} + color indices for q, \tilde{q}

Also the other two generations of quarks/squarks and leptons/sleptons must be added

Fermionic & Bosonic degrees of freedom

- a chiral fermion, e_L 2
- a complex scalar, \tilde{e}_L 2

SUSY associates a complex scalar to each chiral fermion of the SM

- massless gauge boson (γ) 2
- Majorana fermion 2

$$\psi_M = \psi_M^C$$

\downarrow
neutral gauginos

- massive gauge boson (Z) 3
- charged gauge boson (w^\pm) 6
- Dirac fermion (ψ_D)
(charged gauginos) 4

In the SM, the electroweak gauge bosons acquire mass by combining with the Higgs

$$W_\mu^a, B_\mu, H$$

$\tilde{W}, \tilde{B}, \tilde{H}$ mix

$$W^\pm, Z, \gamma, R (H, A, H^\pm)$$

to give 2 charged ψ_D
+ 4 neutral ψ_M

→ 16 d.o.f.

Minimal Supersymmetric Particle Spectrum

Normal Particles

Supersymmetric Partners

| | Weak Int. Eigenstates | Mass Eigenstates |
|----------------------------------|----------------------------|------------------|
| $q = u, d, s$ c, b, t | \tilde{q}_L, \tilde{q}_R | squarks |
| $l = e, \mu, \tau$ | \tilde{l}_L, \tilde{l}_R | selectrons |
| $\nu = \nu_e, \nu_\mu, \nu_\tau$ | $\tilde{\nu}$ | neutrinos |
| g | \tilde{g} | gluino |
| W^\pm | \tilde{W}^\pm | wino |
| H_1^- | \tilde{H}_1^- | higgsinos |
| H_2^+ | \tilde{H}_2^+ | |
| W_μ^a | | |
| γ | $\tilde{\gamma}$ | photino |
| ϕ | $\tilde{\phi}$ | zino |
| Z^0 | \tilde{Z}^0 | higgsinos |
| H_1^0 | \tilde{H}_1^0 | |
| H_2^0 | \tilde{H}_2^0 | |
| | | |

Supersymmetric Lagrangian

- constructed as a fc of a chiral superfield

$$\phi \longrightarrow \begin{array}{l} A \rightarrow \text{scalar} \\ \theta \rightarrow \text{fermion} \end{array} \quad \text{components}$$

$$\phi = A + \theta + F\theta^2$$

Matter fields belong to chiral representations:

Electron Superfield

$$E_L = (\tilde{e}_L, e_L)$$

$$E_R = (\tilde{e}_R^*, (e_R^c))$$

Neutrino Superfield

$$N_L = (\tilde{\nu}_{e_L}, \nu_{e_L})$$

Similarly, quark superfields, Higgs superfields.

Superfield transformation properties under gauge transf.

= to those of their scalar & fermion components

- Scalar & fermion components \rightarrow different transf. properties under a discrete symmetry

$$R\text{-parity: } (-1)^{2S+3B+L}$$

$$\Rightarrow \text{SM particles} \rightarrow R\text{-parity} = 1$$

$$\text{SUSY particles} \rightarrow R\text{-parity} = -1$$

If R-parity conserved:

SUSY (odd) cannot decay into SM (even)

Lightest SUSY Particle STABLE!

Neutral \Rightarrow • \notin as exp. signature • dark matter

i) Self-Interactions of Matter Fields

defined as a fc. of the SUPERPOTENTIAL $P[\phi]$
 — general form in the MSSM:

$$P[A] = \epsilon_{ij} [u H_1^i H_2^j + h_e H_1^i \tilde{L}^j \tilde{E} + h_d H_1^i \tilde{Q}^j \tilde{D} + h_u H_2^i \tilde{Q}^j \tilde{U}]$$

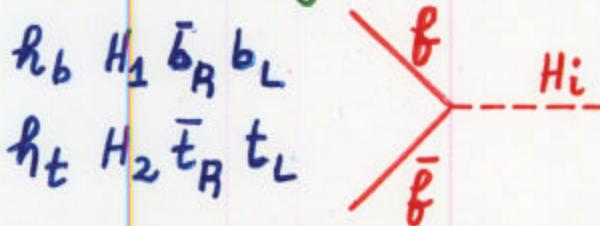
$$\tilde{L}^i = (\tilde{\nu}_e, \tilde{e})_L; \tilde{E} = \tilde{e}_R^*; \tilde{Q}^i = (\tilde{u}, \tilde{d})_L; \tilde{U} = \tilde{u}_R^*; \tilde{D} = \tilde{d}_R^*$$

- defining $P_R = \partial P[A]/\partial A_R$ $P_{ij} = \partial P[A]/\partial A_i \partial A_j$

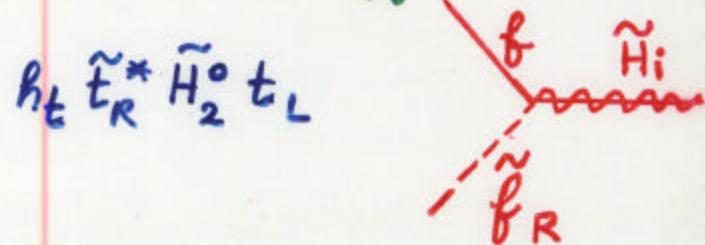
\Rightarrow Yukawa interactions + fermion masses

$$\mathcal{L}_P \rightarrow -\frac{1}{2} (P_{ij} \tilde{q}_i \tilde{q}_j + \text{h.c.})$$

- usual SM Yukawa int.



- stop-top-Higgsino coupl.



$P[A] \rightarrow$ no dep. on H_i^* (only chiral fields)

no $H_1 \tilde{Q} \tilde{U}$ coupling allowed (gauge inv.)

$\Rightarrow H_2$ needed to generate mass to up quarks /

when $\langle H_1 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}$ $\langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$ $v_1 = v \cos \beta$ $\tan \beta = \frac{v_2}{v_1}$

$$m_u = h_u v_2 \quad m_d = h_d v_1 \quad m_e = h_e v_1$$

- Anomaly free Higgsino sector $\int \tilde{q}_i H_1 \rightarrow Y = -1$

Scalar Potential

also determined through the SUPERPOTENTIAL

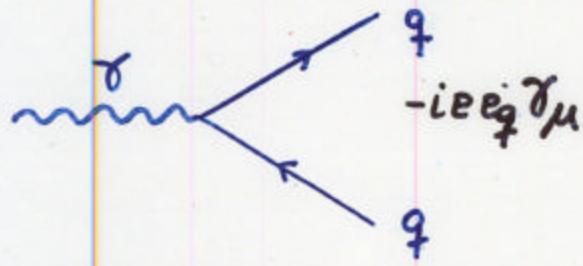
$$V[A] = \sum_k \left| \frac{\partial P[A]}{\partial A_k} \right|^2 + \sum_a \frac{D^a D^a}{2}$$

$$D^a = \sum_A A_i T_{ij}^a A_j g_a$$

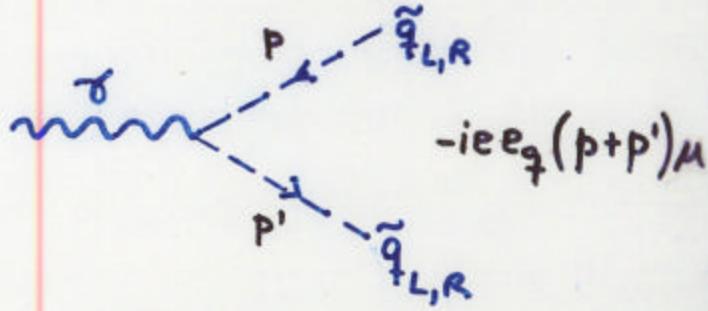
quartic couplings det.
as a fc. of gauge coupl.!
component of the
gauge group generators

ii) Interaction of Gauge and Matter Fields

• usual SM couplings

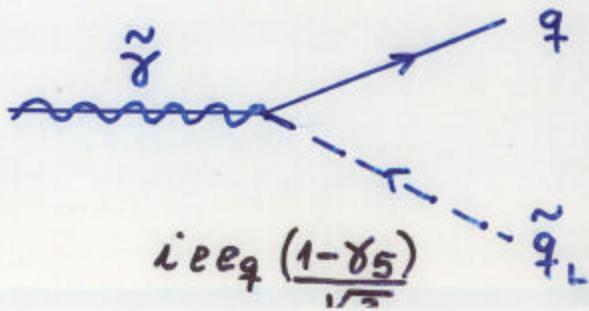
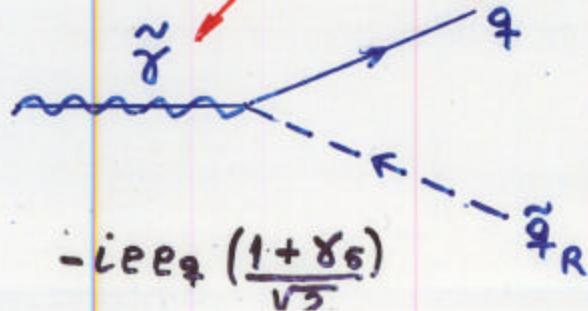


• sparticle-gauge int.



• Novel scalar-gaugino-fermion interaction

→ $A_j^* \tilde{\gamma}_{jk} \tilde{\chi}_k + h.c.$



Self Interaction of Gauge Multiplet

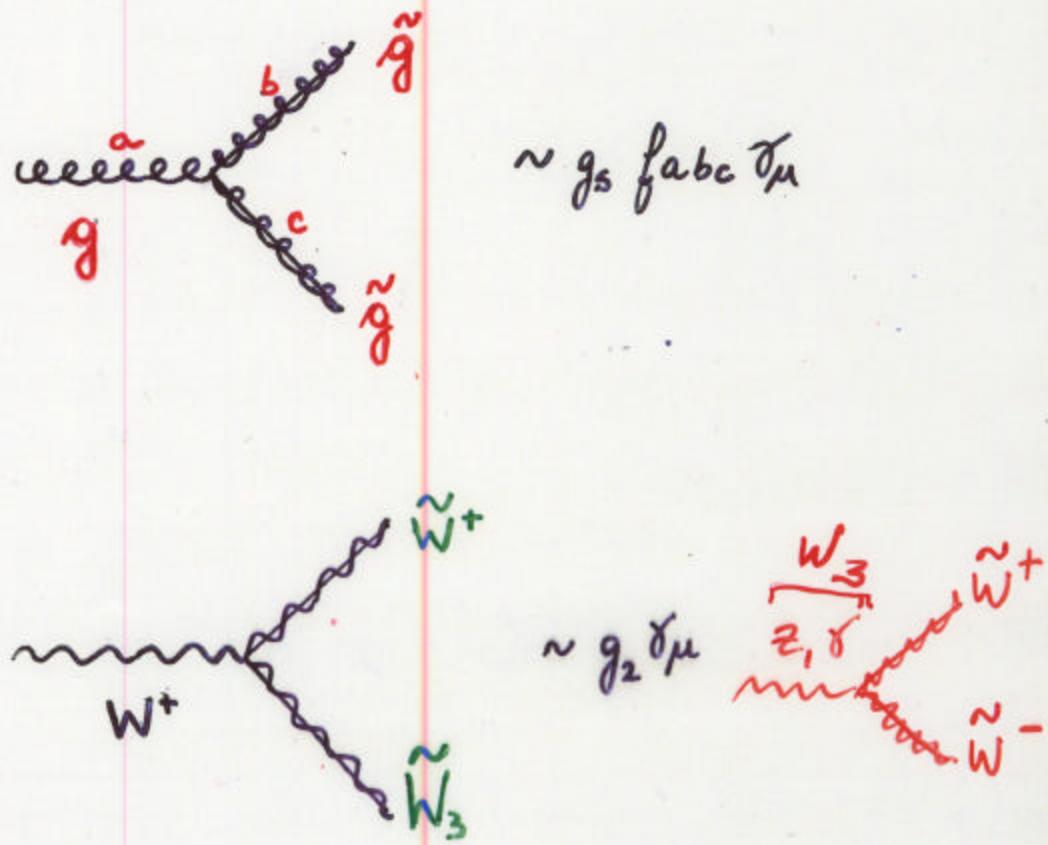
usual three and four gauge boson vertices from kin. term

+ Gaugino-gauge boson interactions

$$-\, i g f_{abc} \bar{\gamma}^a \sigma^\mu \bar{\gamma}^b V_\mu^c$$

structure constant of
gauge group

for example:



- no interaction between the $U(1)$ gaugino and gauge field, $f_{abc} = 0$

If R-Parity Violation

⇒ extra terms in the SUPERPOTENTIAL

$$P[\phi]_R = \tilde{\beta}_{ijk} L_i Q_j D_k + \tilde{\beta}'_{ijk} \underbrace{L_i L_j E_k}_{i \neq j} + \tilde{\beta}''_{ijk} \underbrace{U_i D_j D_R}_{j \neq k}$$

A, 4

LQD & LLE terms violate L number

UDD → violates B number

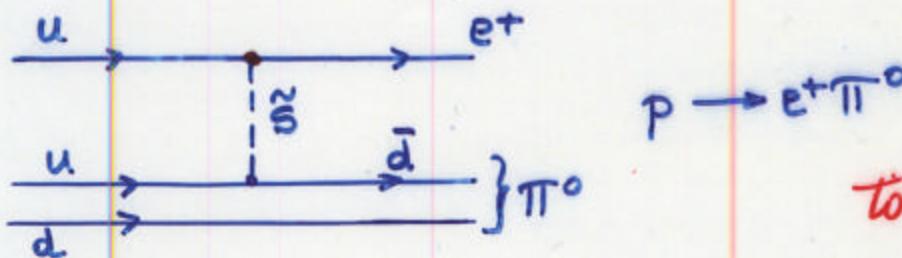
in components: → $\partial^2 P[A]/\partial A_i \partial A_j$ 4i,4j

$$\mathcal{L}_{LLE} = \tilde{\beta}'_{ijk} \left\{ \tilde{v}_L^i e_L^j \tilde{e}_R^k - \tilde{e}_L^i v_L^j \tilde{e}_R^k + \tilde{e}_R^{k*} v_L^i e_L^j \right. \\ \left. + \tilde{v}_L^j e_L^i \tilde{e}_R^k - \tilde{e}_L^j v_L^i \tilde{e}_R^k - \tilde{e}_R^{k*} e_L^i v_L^j \right\} + h.c.$$

$$\mathcal{L}_{LQD} = \tilde{\beta}_{ijk} \left\{ \tilde{v}_L^i d_L^j \bar{d}_R^k - \tilde{e}_L^i u_L^j \bar{d}_R^k + \tilde{d}_L^j v_L^i \bar{d}_R^k - \tilde{u}_L^j e_L^i \bar{d}_R^k \right. \\ \left. + \tilde{d}_R^{k*} v_L^i d_L^j - \tilde{d}_R^{k*} e_L^i u_L^j \right\} + h.c.$$

analogous for \mathcal{L}_{UDD}

- superpartners produced singly, LSP unstable and can be color/charged
- New interactions possible:



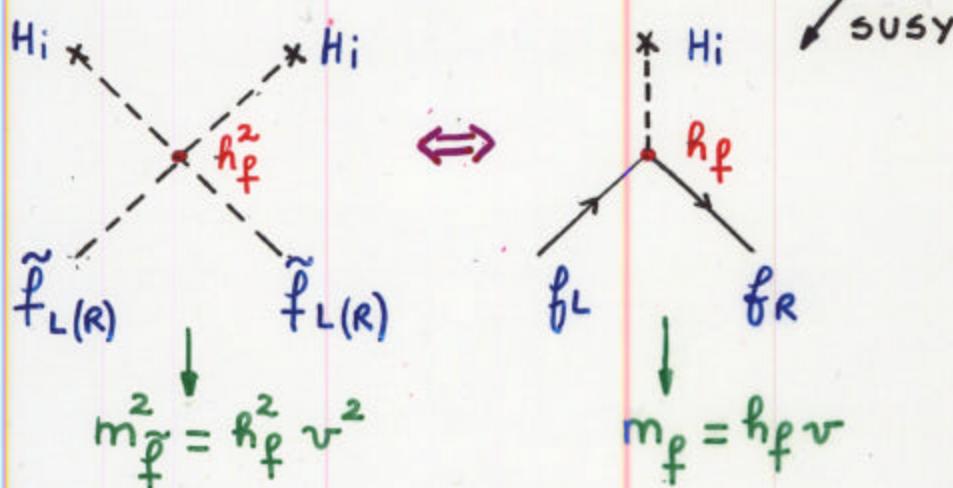
simplest way to
avoid problems
with proton decay
↓
to allow only \cancel{K} or \cancel{B} int.

limits on \mathcal{R} couplings from: proton decay, $\nu u \bar{e}$ scatt, charged current univ., $\Gamma(\pi \rightarrow e\nu)/\Gamma(\pi \rightarrow \mu\nu)$, $\Gamma(\tau \rightarrow e\nu\bar{\nu})/\Gamma(\tau \rightarrow \mu\nu\bar{\nu})$, $\Gamma(\tau \rightarrow \mu\nu\bar{\nu})/\Gamma(\mu \rightarrow e\nu\bar{\nu})$, ...

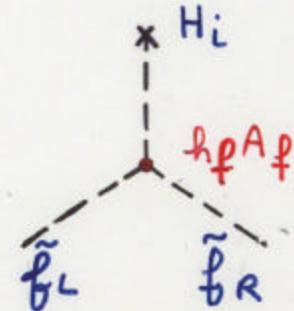
SUSY Breaking

- SUSY must be broken in nature, but SUSY mechanism not well understood yet.
- empirically: add all possible soft SUSY terms
— which preserve cancellation of quad. divergences

- Scalar masses $m_{\tilde{f}}^2$ $\xrightarrow{\text{SUSY}}$ $m_{\tilde{f}}^2 = m_f^2 + m_{f_{\text{SUSY}}}^2$



- Scalar left-right mixing terms A_f :



- Gaugino mass terms $M_i (\beta_i \tilde{\chi}_i + \bar{\beta}_i \tilde{\chi}_i)$

Finite set of soft SUSY param.: $m_{\tilde{f}}^2$, A_f , M_i

\Rightarrow wide range of possible values at high energies
(dep. on SUSY mechanism)

+ Renormalization Group evolution — determine low energy values

= Define SUSY MASS SPECTRUM

$$\mathcal{L}_{\text{SUSY}} = - \sum_{\text{scalars}} m_i^2 A_i^2 - \sum_{\text{gauginos}} M_i (\tilde{\beta}_i \tilde{\beta}_i + \tilde{\bar{\beta}}_i \tilde{\bar{\beta}}_i) \\ + \left(B \mathcal{P}^{(2)}[A] + \sum_k A_k \mathcal{P}_{(k)}^{(3)}[A] + \text{R.c.} \right)$$

where $\mathcal{P}^{(2)}[A] = \mu \epsilon_{ij} H_1^i H_2^j$
 \downarrow
Higgs/Higgsino SUSY mass param.

$B \rightarrow$ extra soft SUSY term, but determined
from cond. of proper EWSB (see next lecture)

$$\sum_k \mathcal{P}_{(k)}^{(3)}[A] A_k = (A_b h_b \tilde{q}^j \tilde{d} H_1^i + A_t h_t \tilde{q}^j \tilde{u} H_2^i \\ + A_\chi h_\chi \tilde{l}^j \tilde{e} H_1^i) \epsilon_{ij}$$

$$\tilde{q} = \begin{pmatrix} \tilde{e}_L \\ \tilde{b}_L \end{pmatrix} \quad \tilde{u} = \tilde{t}_R^* \quad \tilde{l} = \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e} \end{pmatrix}_L \quad \tilde{e} = \tilde{\nu}_R^*$$

trilinear terms, prop. to Yukawa couplings,
 \rightarrow induce Left-right mixing in the squark
sector when Higgs acquire v.e.v.

Mixing prop. to fermion masses

\Rightarrow relevant for 3 gen. only

Soft SUSY parameters in the Stop Sector

$$V_{\text{stop}} \simeq (m_Q^2 + m_t^2) \tilde{t}_L^* \tilde{t}_L + (m_u^2 + m_t^2) \tilde{u}^* \tilde{u}$$

$$+ [A_t h_t H_2 \tilde{E}_L \tilde{u} - \mu h_t H_1 \tilde{t}_L \tilde{u} + \text{h.c.}] + \dots$$

$$\tilde{Q} = \begin{pmatrix} \tilde{t}_L \\ \tilde{b}_L \end{pmatrix} \quad \tilde{u} = \begin{matrix} \tilde{t}_R^* \\ \tilde{u} \end{matrix} \quad m_Q, m_u, A_t \xrightarrow{\text{LR}} \text{soft SUSY}$$

$$M_{\tilde{t}}^2 = \begin{bmatrix} m_Q^2 + m_t^2 + D_{\tilde{E}_L}^2 \\ m_t (A_t - \mu / \tan \beta) \\ m_u^2 + m_t^2 + D_{\tilde{E}_R}^2 \\ m_t (A_t - \mu / \tan \beta) \end{bmatrix}_{RR}$$

$$D_{\tilde{E}_L}^2 = M_Z^2 \cos 2\beta (T_{3t} - Q_t \sin^2 \theta_W)$$

$$D_{\tilde{E}_R}^2 = M_Z^2 \cos 2\beta Q_t \sin^2 \theta_W$$

$T_{3t}, Q_t \rightarrow \text{isospin/electric charge of top.}$

- large mixing due to large Yukawa Coupling
- light stops, $m_{\tilde{t}} < m_t$, may occur if large mixing or $m_u^2 \lesssim 0$ — Electroweak Baryogenesis
- precision measurements — $m_Q^2 \gg m_u^2, m_t^2$

$$m_{\tilde{t}}^2 \simeq m_u^2 + m_t^2 \left(1 - \frac{x_t^2}{m_Q^2} \right)$$

$$m_{\tilde{t}}^2 \simeq m_Q^2 + m_t^2 \left(1 + \frac{x_t^2}{m_Q^2} \right)$$

$$x_t = A_t - \mu / \tan \beta$$

MSSM:

- = minimal SUSY extension of the SM
- = minimal extension of particle content

but, within the MSSM

→ many SUSY scenarios possible ^(see next lecture)

many different boundary conditions for
SUSY param. at the energy scale at which SUSY
is transmitted to observable sector

- Generically, SUSY is broken spontaneously in some
new sector of particles at high energies, when
some of the components of the new (hidden) sector
acquire v.e.v. $\langle F \rangle \rightarrow \text{dim [mass]}^2$

interaction terms between those components and the
MSSM superfields → give rise to δ_{SUSY}

(one can think of Messengers (w/generic mass M) which
couple hidden sector to MSSM sector $\Rightarrow m_{\text{SUSY}} \propto \langle F \rangle / M$)

Renormalization group evolution of SUSY param.

$\Rightarrow \neq$ hierarchy of MSSM particle masses
at low energies

$\Rightarrow \neq$ production & decay patterns
 \neq search strategies at colliders